A Dynamic Data Driven Application System for Real-time Monitoring of Stochastic Damage

E. E. Prudencio, P. T. Bauman, S. V. Williams, D. Faghihi, K. Ravi-Chandar, and J. T. Oden

Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

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Outline

1. Motivation
2. Experimental Data
3. Proposed Model, and its Statistical Calibration
4. State Monitoring with Extended Kalman Filter
5. Ongoing Efforts
1. Motivation
Objective

To develop a proof-of-concept application system that:

- **dynamically** measures electric potential profiles (data) in composite materials,
- uses such data, and damage evolution models, to infer stochastic damage profiles (state), and then
- uses such state to **drive** a decision process (e.g., mesh refinement).
One Experiment, in a Nutshell
Why is the Experiment Useful?

As bar is stretched, damage begins to concentrate in certain points

Abstractly:

• Predict the state while there is no data, at every time step
• When new data arrives, correct the state
• Take data-driven actions based on corrected state
Why is the Experiment Useful?

As bar is stretched, damage begins to concentrate in certain points

Stretch bar slowly, in a controllable way, and:

• simultaneously predict the evolution of damage in the bar;
• at certain instants, make measurements (load, displacements, electric potential, etc);
• use measurements to correct eventual mispredictions;

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Begin simple: interesting enough, computationally tractable, 1D problem
The DDDAS Cycle
## Our Project: Two Phases

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2. Experimental Data
Collected Data: Load and Left Corner Displacement
Collected Data: Stress-Strain History

![Graph showing stress-strain history](image-url)
Used Experimental Data: From 5 Instants
3. Proposed Model, and its Statistical Calibration
One-Dimensional Model, and Equations Involved

Notation:
- $\sigma =$ stress
- $u =$ displacement
- $\epsilon =$ strain
- $D =$ damage
- $C =$ elasticity tensor
- $f =$ body force
- $E =$ Young’s modulus
- $(x_{\text{min}}, x_{\text{max}}) =$ physical domain

Lemaitre/Chaboche Framework:
\[
- \nabla \cdot \left[ (1 - D) C : \epsilon \right] = f
\]
\[
\dot{D} = \frac{\dot{\epsilon}}{\omega' (D)}
\]
\[
\omega (D) = \text{damage model}
\]

One-Dimensional Problem:
\[
- \frac{d}{dx} [(1 - D) E \epsilon] = f
\]
\[
\dot{D} = \frac{\dot{\epsilon}}{\omega' (D)}
\]
\[
u (x_{\text{min}}) = g_1
\]
\[
u (x_{\text{max}}) = g_2
\]

Two Models Within Lemaitre/Chaboche (red to be calibrated):
\[
\omega (D) = \frac{1}{2} E \epsilon_R^2 D \frac{1}{s+1}
\]  
(Krajcinovic)
\[
\omega (D) = \frac{1}{2} \frac{E \epsilon_0^2}{(1 - D)^2}
\]  
(alternative)
Discretization: Finite Elements

\[
\begin{align*}
- \frac{d\sigma^{(k+1)}}{dx} &= 0 \quad \text{in } \Omega, \\
u(x_{\min}) &= g_1^{(k+1)} \\
u(x_{\max}) &= g_2^{(k+1)}.
\end{align*}
\] (1)

1. Given \(u^{(0)}, D^{(0)}\). Step \(k = 0\).
2. Iterate until convergence:
   1. Compute \(u^{(k+1)}\) given loads \(g_1^{(k+1)}\) and \(g_2^{(k+1)}\).
   2. \(\sigma^{(k+1)} = (1 - D^{(k+1)})E\Delta\epsilon\).
   3. Compute \(\psi_d(Y^{(k+1)}, D^{(k+1)})\).
   4. If \(\psi_d(Y^{(k+1)}, D^{(k+1)}) > 0\), \(\sigma^{(k+1)} = \sigma^{(k+1)} - \Delta D E\epsilon^{(k+1)}\).
3. \(k = k + 1\), goto 2.
The Likelihood PDF

measurement + error = unknown reality = model output + model discrepancy
The Likelihood PDF

measurement + error = unknown reality = model output + model discrepancy

\[
d_{N \times 1} + \epsilon_{\text{data}} = \text{unknown reality} = [y(\theta)]_{N \times 1} + \epsilon_{\text{model}}
\]
The Likelihood PDF

measurement + error = unknown reality = model output + model discrepancy

\[ d_{N\times1} + \epsilon_{\text{data}} = \text{unknown reality} = [y(\theta)]_{N\times1} + \epsilon_{\text{model}} \]

\[ \epsilon_{\text{data}} \sim \mathcal{N}(0_{N\times1}, \sigma_{\text{data}}^2 \cdot I_{N\times N}) \]

\[ \epsilon_{\text{model}} \sim \mathcal{N}(0_{N\times1}, \sigma_{\text{model}}^2 \cdot I_{N\times N}) \]

\[ \sigma^2 \equiv \sigma_{\text{data}}^2 + \sigma_{\text{model}}^2 \]
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\[ \sigma^2 \equiv \sigma^2_{\text{data}} + \sigma^2_{\text{model}} \]

\[ \pi_{\text{like}}(d|\theta) \propto \frac{1}{\sqrt{\sigma^2 N}} \cdot \exp \left\{ -\frac{1}{2} \frac{\|d - y(\theta)\|_2^2}{\sigma^2} \right\} \]
### Proposed Model, and its Statistical Calibration

#### Statistical Calibration: Lonestar Platform at TACC

<table>
<thead>
<tr>
<th># simultaneous Markov chains</th>
<th>24</th>
</tr>
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<tbody>
<tr>
<td># processors/chain</td>
<td>1</td>
</tr>
<tr>
<td># processors total</td>
<td>24</td>
</tr>
<tr>
<td># samples total</td>
<td>12,288</td>
</tr>
<tr>
<td>1 likelihood call</td>
<td>1 model call</td>
</tr>
<tr>
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<td>≈ 4 seconds</td>
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Parameters $\theta_1 = \log_{10}(E/E_{ref})$ and $\theta_2 = \log_{10}(s)$
Parameters $\theta_3 = \log_{10}(\epsilon_R)$ and $\theta_4 = \log_{10}(\sigma_{\text{load}})$
4. State Monitoring with Extended Kalman Filter
Prediction and Measurement Steps

Given control $c^{(k+1)}$ and noises $w^{(k)}$ and $v^{(k+1)}$:

$$\theta^{(k+1)} = f^{(k+1)}(\theta^{(k)}, c^{(k+1)}, w^{(k)})$$

$$y^{(k+1)} = g^{(k+1)}(\theta^{(k+1)}, v^{(k+1)})$$

Measure data $d^{(k+1)}$ at $t^{(k+1)}$.

The comparison between the model output $y^{(k+1)}$ and the measurements $d^{(k+1)}$ in the likelihood PDF allows one to statistically update the predicted state $\theta^{(k+1)}$ using Bayes’ formula.
General Bayesian Filtering

Prediction (obtain a prior knowledge of the state):

\[ \pi_{\text{post}}(\theta^{(k)}|d^{(k)}, \ldots, d^{(1)}) \rightarrow \pi_{\text{prior}}(\theta^{(k+1)}|d^{(k)}, \ldots, d^{(1)}) \]

\[
\pi_{\text{prior}}(\theta^{(k+1)}|d^{(k)}, \ldots, d^{(1)}) = \int \pi_{\text{state}}(\theta^{(k+1)}|\theta^{(k)}, d^{(k)}, \ldots, d^{(1)}) \cdot \pi_{\text{post}}(\theta^{(k)}|d^{(k)}, \ldots, d^{(1)}) \, d\theta^{(k)}
\]

Correction (obtain a posterior knowledge of the state):

\[ \pi_{\text{prior}}(\theta^{(k+1)}|d^{(k)}, \ldots, d^{(1)}) \rightarrow \pi_{\text{post}}(\theta^{(k+1)}|d^{(k+1)}, d^{(k)}, \ldots, d^{(1)}) \]

\[
\pi_{\text{post}}(\theta^{(k+1)}|d^{(k+1)}, d^{(k)}, \ldots, d^{(1)}) = \frac{\pi_{\text{like}}(d^{(k+1)}|\theta^{(k+1)}) \cdot \pi_{\text{prior}}(\theta^{(k+1)}|d^{(k)}, \ldots, d^{(1)})}{\pi_{\text{data}}(d^{(k+1)}|d^{(k)}, \ldots, d^{(1)})}.
\]
The Kalman Filter

Known vector $\hat{\theta}^{(0)}$, matrices $A^{(k+1)}$, $B^{(k+1)}$, $H^{(k+1)}$, $P^{(0)}$, $Q^{(k)}$, $R^{(k+1)}$:

\[
\begin{align*}
    f^{(k+1)}(\theta^{(k)}, u^{(k+1)}, w^{(k)}) &= A^{(k+1)} \cdot \theta^{(k)} + B^{(k+1)} \cdot u^{(k+1)} + w^{(k)}, \\
    g^{(k+1)}(\theta^{(k+1)}, v^{(k+1)}) &= H^{(k+1)} \cdot \theta^{(k+1)} + v^{(k+1)}, \\
    \theta^{(0)} &\sim \mathcal{N}(\hat{\theta}^{(0)}, P^{(0)}), \\
    w^{(k)} &\sim \mathcal{N}(0, Q^{(k)}), \text{ and} \\
    v^{(k+1)} &\sim \mathcal{N}(0, R^{(k+1)}).
\end{align*}
\]

\[
\begin{align*}
    \tilde{\theta}^{(k+1)} &= A^{(k+1)} \cdot \hat{\theta}^{(k)} + B^{(k+1)} \cdot u^{(k+1)}, \\
    \tilde{P}^{(k+1)} &= A^{(k+1)} \cdot P^{(k)} \cdot A^{(k+1)^T} + Q^{(k)}, \\
    K^{(k+1)} &= \tilde{P}^{(k+1)} \cdot H^{(k+1)^T} \left( H^{(k+1)} \cdot \tilde{P}^{(k+1)} \cdot H^{(k+1)^T} + R^{(k+1)} \right)^{-1}, \\
    \hat{\theta}^{(k+1)} &= \tilde{\theta}^{(k+1)} + K^{(k+1)} \cdot \left( d^{(k+1)} - H^{(k+1)} \cdot \tilde{\theta}^{(k+1)} \right), \text{ and} \\
    \hat{P}^{(k+1)} &= \left( I - K^{(k+1)} \cdot H^{(k+1)} \right) \cdot \tilde{P}^{(k+1)}.
\end{align*}
\]
The Extended Kalman Filter

Known $\hat{\theta}^{(0)}$, $P^{(0)}$, $Q^{(k)}$, $R^{(k+1)}$, $\forall k \geq 0$:

$\tilde{\theta}^{(k+1)} = f^{(k+1)}(\hat{\theta}^{(k)}, u^{(k+1)}, 0)$,

$\tilde{P}^{(k+1)} = A^{(k+1)} \cdot P^{(k)} \cdot A^{(k+1)T} + W^{(k+1)} \cdot Q \cdot W^{(k+1)T}$,

$K^{(k+1)} = \tilde{P}^{(k+1)} \cdot H^{(k+1)T} \cdot (H^{(k+1)} \cdot \tilde{P}^{(k+1)} \cdot H^{(k+1)T} + V^{(k+1)} \cdot R^{(k+1)} \cdot V^{(k+1)T})^{-1}$,

$\hat{\theta}^{(k+1)} = \tilde{\theta}^{(k+1)} + K^{(k+1)} \cdot (d^{(k+1)} - g^{(k+1)}(\tilde{\theta}^{(k+1)}, 0))$, and

$\hat{P}^{(k+1)} = (I - K^{(k+1)} \cdot H^{(k+1)}) \cdot \tilde{P}^{(k+1)}$,

where

$A_{i,j}^{(k+1)} = \frac{\partial f_i^{(k+1)}}{\partial \theta_j}(\hat{\theta}^{(k+1)}, u^{(k+1)}, 0)$,

$W_{i,j}^{(k+1)} = \frac{\partial f_i^{(k+1)}}{\partial w_j}(\hat{\theta}^{(k+1)}, u^{(k+1)}, 0)$,

$H_{i,j}^{(k+1)} = \frac{\partial g_i^{(k+1)}}{\partial \theta_j}(\tilde{\theta}^{(k+1)}, 0)$,

$V_{i,j}^{(k+1)} = \frac{\partial g_i^{(k+1)}}{\partial v_j}(\tilde{\theta}^{(k+1)}, 0)$. 

Prudencio et al
Evolution of State
5. Ongoing Efforts
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