Optimized Routing of Intelligent, Mobile Sensors for Dynamic, Data-Driven Sampling

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The long-term goal of this project is to provide a control-theoretic framework to enable intelligent, mobile systems to optimally collect sensor-based observations that yield accurate estimates of unknown processes pertaining to Air Force research, including human/vehicle tracking and airborne contaminant release.

The specific research objective is to apply tools from aerospace engineering, specifically nonlinear estimation and control, to design coordinated sampling trajectories that yield the most informative measurements of estimated dynamical and stochastic systems.
The technical approach is to construct a framework for dynamic, data-driven sampling algorithms:

1. Maximize the observability of a nonlinear dynamical system subject to time-varying perturbations;

2. Minimize the uncertainty in the estimate of a nonstationary random process that requires nonuniform sampling; and

3. Experimental work on application-domain problems using small aerial vehicles under automatic control.

The proposed approach incorporates complementary representations of an unknown process: (1) uses a deterministic, model-based parametrization, whereas (2) uses a low-dimensional statistical description.

The approach applies and enables the DDDAS concept in which measurement data is used to update the model description and the updated model is used to guide subsequent measurements.
Relationship to DoD missions

- A DDDAS-based framework is especially relevant to the following situational awareness applications when the unknown process is dynamic and there are incomplete measurements:
  - Relative motion of people or vehicles
  - Propagation of airborne contaminants or pathogens
- The monitoring of airborne contaminant release by multiple aerial vehicles is a capability that this research could enable by optimizing sensor routing for flux estimation through a closed boundary.
Dynamic, data-driven adaptive sampling uses information theory to optimize sensor routes, automatic control to stabilize the desired trajectories, and nonlinear filtering to assimilate date.
1. Observability-based optimization for distributed estimation

A. **Parametrize trajectories** to enable optimization over small state-space and feedback control to stabilize the optimal trajectories.

B. **Evaluate observability** of the estimated flowfield using the *unobservability index* of the flowfield parameters along candidate sampling trajectories.

C. **Recursively estimate** the flowfield parameters by assimilating noisy flow measurements into a Bayesian filter.
Measures of observability

• In linear systems theory, the singular values of the observability gramian determine the relative ease in determining the initial states of a linear system from the outputs generated over a time interval.

• The unobservability index is the reciprocal of the smallest singular value:

\[ \xi \triangleq \frac{1}{\sigma_{\text{min}}}. \]

• Using the observability gramian on a nonlinear system to compute the unobservability index and the estimation condition number requires linearization about an equilibrium point.
Empirical observability gramian

- An alternative is to use the empirical observability gramian, also known as the observability covariance matrix, which maps the input-to-state and state-to-output behavior of a nonlinear system more accurately than the observability gramian found by linearization:

\[
\frac{1}{4\epsilon^2} \int_0^T (y^{+i}(t) - y^{-i}(t))'(y^{+j}(t) - y^{-j}(t)) \, dt
\]

- Requires only the ability to simulate the observed system
- We evaluate the singular values of the empirical observability gramian over a family of candidate sampling trajectories.
Observability-based sampling algorithm

1. Use the current vehicle positions and flowfield estimate to **optimize sampling formation** parameters.

2. Use the flowfield estimate and optimal sampling parameters to **stabilize desired formation** and collect noisy measurements.

3. Use measurements to **update the flowfield estimates** by recursive Bayesian filtering.

**Observability-based Coordinated Sampling**

- Flowfield
- Recursive Bayesian Filter
- Multi-vehicle Control
- Observability Optimization
- Vehicle Dynamics
- Recursive Bayesian Filter

**Fig. 6** A schematic diagram of the observability-based sampling algorithm. A recursive Bayesian filter provides flowfield parameter estimates \( \hat{\Omega} \) from noisy flow measurements \( \tilde{\alpha} \). The estimated flowfield parameters are used to calculate observability optimizing control parameters \( \tilde{\chi} \) that characterize the multi-vehicle sampling formation.

**4.2 Optimization of Formation Position and Radius for Rankine Vortex Sampling**

In this example we optimize over the full parameter space defining the circular sampling family \( \chi^* = (r, |w_0|) \) using the adaptive sampling algorithm in Table 1 and \( N = 5 \) vehicles.

In the first example we assume a moderate speed ground truth flowfield parameterized by \( v_{\text{max}} = 0.5 \), \( r_{\text{max}} = 30 \), and \( \mu = 0.8 \) and in the final example we let \( v_{\text{max}} \) exceed the vehicle speed relative to the flow. The sampling duration is 600 minutes with re-optimization of the control parameters occurring every \( T_{\text{opt}} = 100 \) minutes. Figure 9 shows a snapshot of the observability-based adaptive sampling algorithm at \( t = 20 \) minutes. The initial flowfield parameter estimates are denoted by the magenta dots in the log of the probability density shown in Figures 9(a) and 9(b) along with the ground truth parameters shown by black dots. The initial flowfield parameter estimates provide the basis for the observability analysis shown in Figure 9(c). The white dot shows the minimum unobservability index over the control parameter space \( \chi^* = (r, |w_0|) \) such that vehicles are initially steered to a large circle centered at \( r = 63 \) with radius \( |w_0| = 55 \). The vehicle trajectories are shown in Figure 9(d).

Figure 10 shows a snapshot of the simulated sampling algorithm at \( t = 600 \) minutes. Notice in Figures 10(a) and 10(b) that the parameter estimates have converged toward the ground truth. The observability analysis in Figure 10(c) steers vehicles to a circular formation centered at \( r = r_{\text{max}} = 30 \) with circle radius \( |w_0| = 8 \). It is interesting to note that although the analysis in this sampling algorithm utilizes the estimated flowfield parameters \( \hat{\Omega} \) and includes transient behavior as vehicles converge to their commanded circular formation, there is a similarity to Figure 5(b) in that unobservability seems to be minimized by traversing small circular formations crossing \( r_{\text{max}} \). Figure 10(d) shows the particle tra-
Observability-based optimization of control parameters (i): Rankine vortex

Predicted performance boundaries

Actual performance with boundaries
Bayesian filter

Desired distance from center

Ground truth

Best estimate
Observability-based optimization of control parameters (ii): Wake vortex

Two-aircraft aerodynamic model, with differential pressure sensors

Observability of lead aircraft wake parameters
Wake vortex estimation and control for formation flight
2. Adaptive sampling of nonstationary random processes

A. **Quantify nonstationarity** using spatial and temporal decorrelation scales that may vary in space and time

B. **Perform nonlinear transformation** into a set of coordinates in which uniform sampling is optimal

C. **Achieve uniform coverage** in the transformed coordinates and invert the transformation to recover the optimal sampling trajectories
Evaluating mapping error

- We interpolate measurements of a random spatiotemporal field to generate an estimate of the field and the corresponding error.

- **Mapping error** is the diagonal of the error covariance matrix.

- We adopt a *nonstationary* covariance function:

\[
C(r_i, r_j) = \frac{|\Sigma(r_i)|^{1/4} |\Sigma(r_j)|^{1/4}}{|\Sigma(r_i) + \Sigma(r_j)|^{1/2}} \exp \left[-\frac{1}{2} (r_i - r_j)^T \left( \frac{\Sigma(r_i) + \Sigma(r_j)}{2} \right)^{-1} (r_i - r_j) \right],
\]

where the spatial and temporal decorrelation scales are

\[
\Sigma(r_k) = \text{diag}\{\sigma(\theta_k), \tau(t_k)\}
\]
Mapping error along a closed path

- **Space-time volume**
- **Stationary field**
- **Nonstationary field**
Characterizing sensor swaths in space (one dimension) and time

- Spatially constrained
  - $S_p < 1$
  - $\frac{\tau}{2}$
  - $\frac{\sigma}{2}$

- Temporally constrained
  - $S_p > 1$
  - $\frac{\tau}{2}$
  - $\frac{\sigma}{2}$

- Nonstationary field

For a fleet of spatially and temporally constrained vehicles in a stationary field, each vehicle collects measurements closer together in space for a spatial nonstationarity, each vehicle slows down so the sampling speed has a constant slope in the space-time plane. The algorithm adjusts the speed control algorithm that stabilizes each vehicle to a constant speed vehicle in a nonstationary field. For a constant sampling speed in a stationary field, the variation of the sensor swath that occurs for a constant speed vehicle in a nonstationary field yields an error map represented on the cylinder that encompasses the field.

The error map for two vehicles in a nonstationary field shows the along-track mapping error of two vehicles traveling in a stationary field on a circular trajectory.

The error map for two vehicles illustrates spatial and temporal decorrelation scales for a stationary field. For nonstationary fields, the need for sampling strategies is motivated.
Transformation to render a 2D field locally stationary

Nonlinear map of space-time coordinates from original (r-domain) to new (R-domain) with uniform decorrelation scales.

Square boundary mapping

R-domain \rightarrow r-domain

"Dip" in scale length

Distance is stretched; orientation is preserved
Example solution to uniform coverage problem in R-domain

The R-domain is uniformly covered with no gaps or overlaps using existing Spanning Tree Coverage

R-domain

r-domain

Spanning tree

Closed circuit
Splay control in R-domain provides desired sampling
Online estimation of decorrelation scales (stationary field)

- **Problem**: simultaneously determine the (unknown) spatial and temporal scales of a stationary field and then use those scales to optimally map the field.

- **Solution**: transition from splay in lag space to splay in sampling space using adaptive gain based on uncertainty in the scale estimates.

- **Objective function**:

\[
\min_{\theta} J(\theta) = \sum_{j=1}^{N-1} \left( K \int_{V_j} \rho(q) dq \right) + \left( 1 - K \right) \left( l_j(\theta) - \frac{2\pi}{N} \right)^2
\]

The adaptive gain is based on KL divergence of filter posterior.
Online estimation of decorrelation scales (stationary field)

Spatial scale is estimated using Bayesian filter with Gaussian process likelihood function, and the sampling formation adapts
Summary and ongoing work

- DDDAS research activities in observability- and nonstationarity-based sampling
- Several applications with aerospace vehicles and DoD missions
- Ongoing work on adaptive sampling of nonstationary fields with unknown hyperparameters, experimental work...
3. Experimental applications with aerial vehicles under automatic control

**Testbed:** 18-camera motion capture system & quadrotors equipped with custom onboard flow sensing and control system

**Separated flow in ship air wake** for *autonomous ship landing*

**Downwash model for** *proximity flight*
Motion capture system = $25K
Quadrotor control in wind

- The quadrotor dynamics include the aerodynamic effects of drag, rotor blade flapping, and induced thrust due to translational velocity and external wind fields.
- A dynamic input/output feedback linearization controller that estimates a parametric model of the wind field using a recursive Bayesian filter.
Quadrotor wind estimation using Bayesian filter
Comparison to PID control