An Integrated approach to the Space Situational Awareness Problem

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Outline

- Motivation: Space Situational Awareness
- Introduction: FISST’s Who’s Who and AEGIS-FISST
- AEGIS-FISST Core Idea
- AEGIS-FISST SSA Applications
- Conclusion
## Motivation: Space Situational Awareness

### Space Debris/Objects

<table>
<thead>
<tr>
<th>Natural</th>
<th>Artificial</th>
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<tr>
<td>Meteoroids &amp; micrometeoroids</td>
<td>Active and inactive payloads, rocket stages, lens caps, solid rocket exhaust particles, explosive bolts, etc.</td>
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<td>A few millimeters in diameter</td>
<td>Meters to less than a cm in diameter</td>
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<td>Very high speeds --20 km/s avg., up to 70 km/s (sun orbits)</td>
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### Motivation

#### Space Situational Awareness

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<td>Diameter &gt; 10 cm</td>
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<td>~22,000 objects</td>
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Risk mitigated through shielding against “debris clouds”
## Motivation

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Risk mitigated through safe target placement, control, estimation of debris/object trajectory, and sensor tasking and control
Introduction: FISST

Finite Set Statistics and SSA

- FISST: a **hybrid** estimation technique applied to aerial, terrestrial and naval joint **detection** and **tracking** problems
- Developing FISST-based estimator for solving **joint search**, detection, classification & tracking (JSDCT=SA) problem:
  - Detection, classification, data association (discrete) vs. tracking (continuous)
  - Consistently exploits information interdependence of object’s existence, class and track
- FISST provides information metric for sensor tasking: detection, classification & tracking information drives sensor tasking. Q: Which data maximizes awareness?
Introduction: Bayesian FISST-Based filtering

Search environment at time $t_k$

$X_k = \{x_1, k\}$ (Clutter source)

$(\text{True state})$

$(\text{Imperfect})$ sensing process

Estimation

$\hat{X}_{k+1} \sim (\hat{n}, \hat{x}_i)$

Posterior at $t_{k+1}$

$\hat{X}_{k+1} \sim (\hat{n}, \hat{x}_i)$

Propagation step

AEGIS

Sensor model

$\mathbf{f}(\mathbf{z}_k | \mathbf{x}_{1,k})$ $\mathbf{g}(\mathbf{z}_k | \mathbf{x}_F)$

Update step

FISST

Propagated prior

$\mathbf{f}_{k+1|k+1}(\mathbf{X}_{k+1})$

Measurements

$Z_k = \{z_{1,k}, z_{2,k}\}$

$z_{1,k} \sim \mathbf{f}(z_{1,k} | x_{1,k}), \ z_{2,k} \sim \mathbf{g}(z_{2,k} | x_c)$

or

$z_{2,k} \sim \mathbf{f}(z_{2,k} | x_{1,k}), \ z_{1,k} \sim \mathbf{g}(z_{1,k} | x_c)$
AEGIS-FISST

Basic Idea

- Recall e.g. multi-target density (2-target hypothesis)
  \[ f(\{x_1, x_2\}) = p_2 \cdot (f^1(x_1)f^2(x_2) + f^2(x_1)f^1(x_2)) \]

- If
  - all conventional prior pdfs are Gaussian mixtures (GMs)
  - all conventional Markov transitional pdfs are GMs
  - all conventional measurement likelihoods are GMs
  - (no restrictions on clutter generation model)

- Then
  - all prior, propagated, and posterior multi-target pdfs are sums of Gaussians;
  - closed form solution to FISST-based filtering

- GM Propagator: AEGIS
Sensor Tasking

- Cauchy-Schwarz information divergence
  \[ D_{SC}(h(X)|g(X)) = \frac{1}{2} \log \left[ \frac{\int h^2(X)\delta X \left( \int g^2(X)\delta X \right)}{\left( \int h(X)g(X)\delta X \right)^2} \right] \]

- Cauchy-Schwarz information gain
  \[ G_{SC}(h(X)|g(X); x^s_{t+1}) = \frac{1}{2} \int f_{k+1}(Z_{k+1}|Z^{(k)}; x^s_{t+1}) \log \left[ \frac{\int h^2(X)\delta X \left( \int g^2(X)\delta X \right)}{\left( \int h(X)g(X)\delta X \right)^2} \right] \delta Z_{k+1} \]

- Optimal sensor allocation
  \[ x^s_{t+1}^{*} = \arg \max_{x^s_{t+1}} G_{SC}(h(X)|g(X); x^s_{t+1}) \]

- Can compute CS divergence in closed-form
- Can compute gain using Monte-Carlo integration
- Optimization: Information Space Receding Horizon Control (ISRHC)
Sensor Allocation for JDT Using ISRHC

- At most two objects (in the plane) --two actually exist
- Apogee: 6,878.10 km, $e = 0.01$, $M$: 23, 22 degrees
- Ground sensor initialized at 30 degrees from inertial x-axis
- Sensor field-of-view: 10 degrees
- A finite set of sensor actions:
  - Look directions with fixed field-of-view:
    
    \{-40,-30,-20,-10,0,10,20,30,40\} deg from local vertical
  - False alarm rate: 0.1; uniformly distributed inside the FOV
  - A Measurement is attempted every 5 seconds
Sensor Allocation for JDT Using ISRHC

\[ \mu_0^1 = (6, 190 \text{ km}, 2, 6855 \text{ km}, -3.0 \text{ km/s}, 7.1 \text{ km/s}) \]

\[ \mu_0^2 = (6, 236 \text{ km}, 2, 575 \text{ km}, -2.9 \text{ km/s}, 7.1 \text{ km/s}) \]

\[
P_0^1 = \begin{bmatrix}
1 \text{ km}^2 & 0 & 0 & 0 \\
0 & 1 \text{ km}^2 & 0 & 0 \\
0 & 0 & 1 \text{ m}^2/\text{s}^2 & 0 \\
0 & 0 & 0 & 1 \text{ m}^2/\text{s}^2
\end{bmatrix}
\]

\[
P_0^2 = \begin{bmatrix}
10,000 \text{ km}^2 & 0 & 0 & 0 \\
0 & 10,000 \text{ km}^2 & 0 & 0 \\
0 & 0 & 0.01 \text{ km}^2/\text{s}^2 & 0 \\
0 & 0 & 0 & 0.01 \text{ km}^2/\text{s}^2
\end{bmatrix}
\]
Sensor Allocation for JDT Using ISRHC

- Initial prior of object existence: $p_0=0.1$, $p_1=0.1$, $p_2=0.8$
- Prior mean state:

  $\mu_0^1 = (6, 190 \text{ km}, 2, 6855 \text{ km}, -3.0 \text{ km/s}, 7.1 \text{ km/s})$
  
  $\mu_0^2 = (6, 236 \text{ km}, 2, 575 \text{ km}, -2.9 \text{ km/s}, 7.1 \text{ km/s})$

- Prior covariance: $P_0^1 = \begin{bmatrix} 1 \text{ km}^2 & 0 & 0 & 0 \\ 0 & 1 \text{ km}^2 & 0 & 0 \\ 0 & 0 & 1 \text{ m}^2/\text{s}^2 & 0 \\ 0 & 0 & 0 & 1 \text{ m}^2/\text{s}^2 \end{bmatrix}$

  $P_0^2 = \begin{bmatrix} 10,000 \text{ km}^2 & 0 & 0 & 0 \\ 0 & 10,000 \text{ km}^2 & 0 & 0 \\ 0 & 0 & 0.01 \text{ km}^2/\text{s}^2 & 0 \\ 0 & 0 & 0 & 0.01 \text{ km}^2/\text{s}^2 \end{bmatrix}$
Sensor Allocation for JDT Using ISRHC
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Conclusion

- AEGIS-FISST utilizes exactness of FISST by employing an AGMM for propagation (vs. first moment approaches)
- Demonstrated the use of AEGIS-FISST on various SSA joint sensor allocation, detection, classification, tracking problems
- Current work at ADS:
  - Extension to the N-target SSA problem
  - Implementing AGMM/PF-FISST in Java and high-performance/parallel computing environments
  - Comparison to other hybrid estimation and sensor allocation techniques
  - Ultimate Goal: Holistic sensor allocation for JSDCT
Questions
Introduction: Bayesian Filtering

Target Assumed Detected

$\mathbf{x}_k$
(True state)

(Imperfect) sensing process

ML Estimation

$\left(\hat{\mathbf{x}}_k, \hat{\mathbf{P}}_k\right)$

Posterior at $t_{k+1}$

$f_{k+1|k+1}(\mathbf{x}_{k+1})$

Sensor model

$f_k(\mathbf{z}_k|\mathbf{x}_k)$

Update step

Bayes Law

Propagated prior

Prior at $t_{k+1}$

$\mathbf{t}_{k+1} \mapsto \mathbf{t}_k$

Measurement

$\mathbf{z}_k \sim f_k(\mathbf{z}_k|\mathbf{x}_k)$
FISST – Based Multi – Target PDSFs

(& other approximations)

(& other approximations)
Introduction: FISST’s Who’s Who (GM World)

- FISST – Based Multi – Target PDSFs
- First Moment Approximation
- PHD Method

GM Approximation (& other approximations)

GM – FISST

AEGIS

AEGIS – FISST

GM – PHD

AEGIS

AEGIS – PHD

AEGIS

AEGIS – FISST

PHD Method
AEGIS-FISST Applications

Joint Detection & Tracking

- Red: Single return measurement of target only
- Blue: Single return measurement of clutter only
- Black: Two return measurement of target and clutter
AEGIS-FISST Applications

Joint Detection & Tracking

- **Red**: Single return measurement of target only
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Graphs showing:
- Number of Gaussian Mixture Components vs. Time (hours)
- Information Divergence vs. Time (hours)
AEGIS-FISST Applications
AEGIS-FISST Applications

Sensor Allocation for Joint Detection and Tracking

- At most a single object (in the plane)
AEGIS-FISST Applications

Sensor Allocation for Joint Detection and Tracking

• At most a single object (in the plane)
• Apogee: 6,878.10 km, eccentricity: 0.01
Sensor Allocation for Joint Detection and Tracking

- At most a single object (in the plane)
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AEGIS-FISST Applications

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- First flyover is 7.5 minutes from initial time
AEGIS-FISST Applications

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- Sensor field-of-view: 5 degrees
AEGIS-FISST Applications

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- A finite set of sensor actions:

[InfoFusion 2012b, SFM 2013a]
AEGIS-FISST Applications

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AEGIS-FISST Applications

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  - Look directions with fixed field-of-view: 
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  - False alarm rate: 0.2; spatially uniformly distributed inside the field-of-view

[InfoFusion 2012b, SFM 2013a]
AEGIS-FISST Applications

Sensor Allocation for Joint Detection and Tracking

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A Measurement is attempted every 5 seconds
AEGIS-FISST Applications

Sensor Allocation for Joint Detection and Tracking

- Initial prior of object existence: 0.5
Initial prior of object existence: 0.5

Prior mean state:

$$\mu_0^1 = (6, 741.0 \text{ km}, 0 \text{ km}, 0 \text{ m/s}, 7.7 \text{ km/s})$$
Sensor Allocation for Joint Detection and Tracking

- Initial prior of object existence: 0.5
- Prior mean state:
  \[ \mu_0^1 = (6, 741.0 \text{ km}, 0 \text{ km}, 0 \text{ m/s}, 7.7 \text{ km/s}) \]
- Prior covariance:
  \[ P_0^1 = \begin{bmatrix}
  10,000 \text{ km}^2 & 0 & 0 & 0 & 0 \\
  0 & 10,000 \text{ km}^2 & 0 & 0 & 0 \\
  0 & 0 & 1 \text{ m}^2/\text{s}^2 & 0 & 0 \\
  0 & 0 & 0 & 1 \text{ m}^2/\text{s}^2 & 0 \\
  0 & 0 & 0 & 0 & 1 \text{ m}^2/\text{s}^2 \\
\end{bmatrix} \]
Sensor Allocation for JDT Using ISRHC
Sensor Allocation for JDT Using ISRHC

(a) Object in view (1 = yes, 0 = no)

(b) Probability of existence
Sensor Allocation for JDT Using ISRHC
Sensor Allocation for JDT Using ISRHC

(c) Optimal look direction and relative angle to object  
(d) Optimal look direction and relative angle to object
Sensor Allocation for JDT Using ISRHC
Sensor Allocation for JDT Using ISRHC

(e) True divergence $D_{SC}$

(f) Optimal Expected Gain
AEGIS-FISST Applications

Sensor Allocation for Joint Detection and Tracking

(a) $x$-position tracking error

(b) $y$-position tracking error
Sensor: Optical, measuring angle only

\[ z_k = \theta_M + v_k \]

Nonzero detection probability (geometry- & SNR-dependent) inside FOV

Clutter is Poisson distributed with uniform distribution within FOV
AEGIS-FISST Applications
Multi-Target Tracking & Data Association w/ Clutter

Simulation Parameters

- 3 objects in orbit
  
  \[ a = 42164.173 \text{ [km]}, \ e = 0, \ \omega = 0 \text{ [deg]}, \ M = 0 \text{ [deg]} \]

  \[ a = 42164.173 \text{ [km]}, \ e = 0, \ \omega = 0 \text{ [deg]}, \ M = 0.5 \text{ [deg]} \]

  \[ a = 42264.173 \text{ [km]}, \ e = 0.01, \ \omega = 0 \text{ [deg]}, \ M = 2.2 \text{ [deg]} \]

- Initial uncertainty: 1 km (pos), 0.1 m/s (vel), 18 hours prior to 1st measurement

- Measurements taken every 30 seconds over two nights for half an hour centered around midnight

- Zero mean, (2 arcsec)\(^2\) covariance angle measurement

- FOV half-angle: 2 deg

- Detection probability: 0.8 inside FOV

- Clutter Poisson mean is 5
AEGIS-FISST Applications

Multi-Target Tracking & Data Association w/ Clutter

Sensor Returns, $Z_k$ [deg]

Time Past Epoch [day]

Number of Sensor Returns, $|Z_k|

Time Past Epoch [day]

Position Error [km]

Time Past Epoch [day]

Object #1
Object #2
Object #3

(a) Position error

Velocity Error [m/s]

Time Past Epoch [day]

Object #1
Object #2
Object #3

(b) Velocity error